Effect of Layers Position on Fracture Toughness of Functionally Graded Steels in Crack Divider Configuration

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In the present study, fracture toughness of functionally graded steels in crack divider configuration has been modeled. By utilizing plain carbon and austenitic stainless steels slices with various thicknesses and arrangements as electroslag remelting electrodes, functionally graded steels were produced. The fracture toughness of the functionally graded steels in crack divider configuration has been found to depend on the composites’ type together with the volume fraction and the position of the containing phases. According to the area under stress-strain curve of each layer in the functionally graded steels, a mathematical model has been presented for predicting fracture toughness of composites by using the rule of mixtures. The fracture toughness of each layer has been modified according to the position of that layer where for the edge layers, net plane stress condition was supposed and for the central layers, net plane strain condition was presumed. There is a good agreement between experimental results and those acquired from the analytical model.

KEY WORDS: Fracture toughness; Modeling; Functionally graded steel; Crack divider configuration; Position of the containing phases

1. Introduction

Development of functionally graded materials (FGMs) is of technological importance which encourages the researchers to produce applicable FGMs with the lowest residual stresses. There are a wide variety of FGMs where variations in elastic constants appear. But a main group of FGMs are those in which variations in strength emerge. In fact, in all structures consisting of multi-phase materials, composites, or functionally graded materials, strength variations are inherent. Therefore, considerations of these group of FGMs are inevitable[1].

The first published experimental evidence that a gradient in yield stress influences the behavior of cracks was performed by Suresh et al[1]. They conducted fatigue experiments on an explosion clad bimaterial consisting of a ferritic and an austenitic steels. A practical application of this experimental finding has been reported in Suresh et al.[2]. Koledkin[3] provided an analytical model to explain why gradients in yield stress affects the crack growth behavior. It was demonstrated that a yield stress gradient induces an additional term of the crack driving force, which leads to an increase or decrease of the effective crack driving force. Becker et al.[4] modeled fracture toughness measured by SE(T) specimen with cracks perpendicular to and along the strength gradient and homogeneous Young modulus using Weibull statistics. Bezensek and Hancock[5] studied the fracture toughness of functionally graded steels produced by laser welding. A method of creating a functionally graded structural member by transforming its material at cryogenic temperatures has been presented by Skoczen[6]. The technique

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consisted of imposing on a stainless steel bar kinematically controlled torsion until the phase transformation threshold is reached and the material starts transforming itself close to the outside radius of the bar.

Most of the work conducted on FGMs with variations in strength has been focused on production methods numerical and theoretical procedures are rarely presented for these materials. More experimental, theoretical and numerical studies need to be presented especially on mechanical properties of FGMs with variations in strength.

Functionally graded steels (FGSs) with strength gradient were produced from austenitic stainless steel and plain carbon steel by using electro slag refining (ESR)\(^7\). By selecting appropriate arrangement and thickness of the original ferritic steel (\(\dot{\alpha}\)) and original austenitic steel (\(\dot{\gamma}\)) as electrodes, it is possible to obtain composites with several layers consisting of ferrite, austenite, bainite and martensite;

\[
(\dot{\alpha}\dot{\gamma})_{el} \xrightarrow{R} (\alpha\beta\gamma)_{com}
\]

\[
(\dot{\gamma}\dot{\alpha}\dot{\gamma})_{el} \xrightarrow{R} (\gamma\gamma\gamma)_{com}
\]

\[
(\dot{\gamma}\dot{\alpha}\dot{\gamma})_{el} \xrightarrow{R} (\gamma\gamma\gamma)_{com}
\]

\[
(\dot{\alpha}\dot{\gamma}\dot{\alpha})_{el} \xrightarrow{R} (\alpha\beta\gamma\beta\alpha)_{com}
\]

\[
(\dot{\alpha}\dot{\gamma}\dot{\alpha})_{el} \xrightarrow{R} (\alpha\beta\gamma\alpha\gamma)_{com}
\]

where \(\alpha, \beta, \gamma\) and M are ferrite, bainite, austenite and martensite phases in final composite, respectively, el is electrode, com is composite, and R is remelting.

As alloying elements such as carbon, chromium and nickel atoms diffuse, alternating regions with different transformation characteristics are created. The diffusing atoms individually or together stabilize different phases such as bainite or martensite. Thicknesses of bainitic and martensitic layers depend on the thickness of the primary electrodes and process variables\(^7\).

Transformation characteristics\(^7\), tensile properties\(^8\) and Charpy impact energy in crack divider\(^9\) and crack arrester\(^10\) configurations of FGSs have previously been investigated experimentally and theoretically. Previously, fracture toughness of functionally graded steels has been modeled\(^11\) and modified\(^12\). Although fracture toughness of the composites was predicted in those works\(^11,12\), the effects of the position of the containing layers was remained un-known. In this work, fracture toughness, \(J_{IC}\), of FGSs has been investigated in crack divider configuration and a mathematical model correlating fracture toughness of FGSs to the sum of the fracture toughness of each layer using the rule of mixtures has been proposed. Fracture toughness of each layer has been modified according to the position of that layer (\(i.e.\) for the edge layers, plane stress condition and for the central layers, plane strain condition has been presumed.).

2. Experimental

To form FGSs, as stated in the previous works\(^7–12\), a miniature ESR apparatus was used. The slag consumed was a mixture of 20% CaO, 20% Al\(_2\)O\(_3\) and 60% CaF\(_2\). The original ferritic and austenitic steels employed as electrodes were commercial type AISI 1020 and AISI 316 steels. Their chemical composition is given in Table 1.

| Table 1 Chemical composition of original alpha and gamma steels (wt%) |
|-----------------|-----|-----|-----|-----|-----|-----|
|                 | C   | Si  | Mn  | P   | S   | Cr | Ni  |
| Original austenitic steel | 0.07 | 0.045 | 0.03 | 18.15 | 9.11 |
| Original ferritic steel     | 0.2 | 0.3 | 0.2 | 0.05 | –   | –  |

Different arrangements of ferritic and austenitic steel slices in the form of 2- and 3-piece electrode were spot welded for remelting. The height of each slice in the primary 2-piece \(\dot{\alpha}\dot{\gamma}\) electrode was 150 mm. For 3-piece \(\dot{\gamma}\dot{\alpha}\dot{\gamma}\) electrode, the height of the middle slice was 25 mm and that of neighboring slices was 137.5 mm.

Remelting was done under a constant power supply of 16 kVA. After remelting, the composite ingots were hot-pressed at 980°C down to the thickness of 30 mm followed by air-cooling. From the produced \(\alpha\beta\gamma\) composite ingot, eight series of fracture toughness specimen were produced in some manner that the bainite intermediate layers was placed at different positions with respect to the specimen’s edges; in four series of specimens the
bainite layer was displaced towards the alpha region and in the other four towards the gamma region. From \( \gamma M \gamma \) composite ingot, due to its symmetric configuration, only four series of fracture toughness specimens with different positions of martensite layer were produced.

Because of limitation of specimen dimensions, fracture toughness measurement in terms of \( K_{IC} \) was not possible. Thus, fracture toughness in terms of \( J_{IC} \) test was carried out on specimens at 18\(^\circ\)C. Specimens’ dimension was in accordance to the ASTM E1820\cite{13} and it is illustrated in Fig. 1(a). Three-point bend specimens were used to investigate the fracture toughness of the composites. The notch depth was 8 mm and a 2 mm fatigue pre-crack was introduced at the end of notch root by applying 3-point cyclic loading under frequency of 10 Hz. The single specimen method using unloading-reloading procedure was performed. After loading a specimen, a partial unloading up to 10 percent of the maximum load was applied and then the specimen was reloaded up to the maximum load. Calculation of the maximum load is given in the ASTM E1820\cite{13} standard and according to the previous work\cite{8} the yield stress of the specimen was determined by the rule of mixtures. Fracture toughness of FGS specimens with the starter crack normal to the graded layers (crack divider configuration as illustrated in Fig. 1(b)) was measured.

Fracture toughness and tensile properties of as-received ferritic and austenitic steels which were annealed at 980\(^\circ\)C and then were air-cooled and fracture toughness and tensile properties of single-phase bainite and martensite with composition and mechanical properties analogous to the bainite and martensite layers was also measured. The production method of single-phase specimens with chemical composition and mechanical properties identical to the bainitic and martensitic layers was similar to the previous work\cite{8–12}. To do this, fracture toughness and tensile test specimens with the same composition and mechanical properties as that of the bainitic and martensitic layers were produced. Initially, the average chemical composition of the bainite and martensite layers was obtained (Table 2). Afterward, bainitic and martensitic samples with chemical composition in accordance to the average chemical composition of single-phase bainitic and martensitic specimens were produced by means of a vacuum induction furnace. Similar to the primary composites, the hot-pressing process was carried out at 980\(^\circ\)C, followed by air cooling. Through trial and error (i.e., confirming the chemical composition and changing the cooling rate), the samples with the nearest hardness to single-phase bainitic and martensitic specimens were selected and fracture toughness and tensile test specimens were made from the bainitic and martensitic samples. Fracture toughness and tensile test results of single-phase bainite and martensite layers produced from the sample are shown in Table 3.

Tensile tests were done under an extension rate of 0.1 mm/s. Tensile specimens dimensions are in accordance

<table>
<thead>
<tr>
<th>Specimen studied</th>
<th>Cr</th>
<th>Ni</th>
<th>C</th>
<th>Si</th>
<th>Mn</th>
<th>S</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single phase bainite</td>
<td>15.4</td>
<td>7.2</td>
<td>0.12</td>
<td>0.8</td>
<td>1.8</td>
<td>0.03</td>
<td>0.04</td>
</tr>
<tr>
<td>Bainite specimen produced from the sample</td>
<td>14.7</td>
<td>7.15</td>
<td>0.13</td>
<td>0.85</td>
<td>1.9</td>
<td>0.032</td>
<td>0.045</td>
</tr>
<tr>
<td>Single phase martensite</td>
<td>7.3</td>
<td>3.2</td>
<td>0.19</td>
<td>0.39</td>
<td>0.3</td>
<td>0.04</td>
<td>0.05</td>
</tr>
<tr>
<td>Martensite specimen produced from the sample</td>
<td>7.38</td>
<td>3.14</td>
<td>0.21</td>
<td>0.28</td>
<td>0.28</td>
<td>0.033</td>
<td>0.055</td>
</tr>
</tbody>
</table>

Fracture toughness and tensile test results of the original ferrite, original austenite, single-phase bainite and single-phase martensite

<table>
<thead>
<tr>
<th>Specimen studied</th>
<th>Yield stress /MPa</th>
<th>Tensile stress /MPa</th>
<th>Fracture toughness, ( J_{IC}/(kJ/m^2) )</th>
<th>Area under stress-strain curve (MPa per unit volume)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original ferrite (( \dot{\alpha} ))</td>
<td>245</td>
<td>425</td>
<td>9</td>
<td>71.4</td>
</tr>
<tr>
<td>Original austenite (( \dot{\gamma} ))</td>
<td>200</td>
<td>480</td>
<td>50</td>
<td>155.6</td>
</tr>
<tr>
<td>Bainite specimen produced from the sample</td>
<td>1025</td>
<td>1125</td>
<td>29</td>
<td>120.2</td>
</tr>
<tr>
<td>Martensite specimen produced from the sample</td>
<td>1440</td>
<td>1482</td>
<td>0.16</td>
<td>12.4</td>
</tr>
</tbody>
</table>
with the ASTM E8\cite{14} standard, as shown in Fig. 2.

For metallographic examination, the plates were sliced, ground, polished, and etched in a “Kalling” solution and 1 pct “Nital”.

Vickers microhardness tests were carried out using 100 g weight to evaluate microhardness profile and to verify the location of bainite or martensite intermediate layers.

3. Results and Discussion

Figures 3(a) and (b) illustrate Vickers microhardness profile of $\alpha\beta\gamma$ and $\gamma\gamma\gamma$ composites. As reported by previous work\cite{7-12}, metallographic studies from cross section of the produced composites show that the new stabilized phases in all composites produced under similar condition have similar features (Fig. 4); the thickness of martensite layer is 1.5 mm and that of bainite is 0.6 mm which was verified by Vickers microhardness examination as shown in Fig. 4.

A typical fracture resistance, $J-R$, curve of the studied composites is shown in Fig. 5. Fracture toughness, $J_{IC}$, of the functionally graded steels which was determined by the offset line with 0.2 mm crack extension is shown in Table 4 and that of single-phase specimens is shown in Table 3.

Table 4 shows improvement of fracture toughness in $\alpha\beta\gamma$ composites with respect to the AISI 1020 steel.
The improvement of fracture toughness in αβγ composite is due to bainite formation with relatively high fracture toughness with respect to the AISI 1020 steel. Previous work\cite{10} shows that in crack divider configuration Charpy impact energy obeys the rule of mixtures with respect to the constituent alpha, bainite and gamma phases. Similarly in this work it has been assumed that fracture toughness of αβγ composite obeys the rule of mixtures.

Table 4 shows that the fracture toughness of γMγ composites is higher than AISI 1020 steel and lower than αβγ composite. Although the thickness of austenitic phase in γMγ is larger than that of αβγ composite, formation of martensite layer causes a considerable reduction in fracture toughness of γMγ. However, the presence of austenite phase with high fracture toughness recovers the fracture toughness of γMγ composite. Charpy impact energy of γMγ composite in crack divider configuration determined by the rule of mixtures\cite{9,10} shows a relatively large deviation from the experimental data. It has been argued that the sharp gradient of impact energy adjacent to the newly formed martensite layer inflicts discrepancy in the results\cite{9,10}. However, in the present work, the rule of mixtures has been examined for fracture toughness prediction of γMγ composite.

Hutchinson\cite{15}, and Rice and Rosengeren\cite{16} separately deduced the proportionality between J and the area under stress-strain curve in power-law work-harden materials. Assuming that αβγ and γMγ composites are composed of graded α and/or γ regions (with several layers) together with bainite or martensite layers. JIC of each layer in graded regions could be determined from stress-strain curve of the corresponding layer. JIC of the composite may be the sum of JIC of the composite's layers based on the rule of mixtures.

To model fracture toughness of αβγ functionally graded steel with the bainite layer at the middle of the specimen, the composite may be considered as a combination of ferritic and austenitic regions with mα and nγ layers, respectively in addition to the bainite layer (Fig. 3(a)). It is assumed that the area under stress-strain curve of each layer is related to its fracture toughness. To model the fracture toughness of αβγ composite, it has been presumed that the fracture toughness (JIC) of the specimen is related to the fracture toughness of the individual layer (i.e. the area under stress-strain curve of each layer) combined with the fracture toughness of the bainite layer according to the rule of mixtures.

Stress-strain curves of original ferritic steel, bainite layer and original austenitic steel have been plotted in Fig. 6.\cite{8,10}. Stress-strain curve of each layer in α and γ graded regions may be represented by several curves according to the number of layers and could be represented as α1, α2, . . . , αm in alpha region and as γ1, γ2, . . . , γn in gamma region. It is observed that the corresponding area under stress-strain curve of each layer increases from α layer (i.e. the first layer of the graded alpha region, α1) towards γ layer (i.e. the last layer of the graded gamma region, γn). It is assumed that the yield stress of each layer is proportional to the Vickers microhardness of that layer\cite{8}. Therefore, the yield stress of each layer in α and γ regions should also obey the hardnes pattern. The yield stress of each layer may be related to the Vickers microhardness of that layer as\cite{8};

\[
\sigma_y(\alpha_i) = \frac{\sigma_y(\beta) - \sigma_y(\gamma)}{VH(\beta) - VH(\gamma)} VH(\alpha_i) + \frac{\sigma_y(\alpha_i) VH(\beta) - \sigma_y(\gamma_i) VH(\gamma)}{VH(\beta) - VH(\gamma)}
\]

\[
\sigma_y(\gamma_i) = \frac{\sigma_y(\beta) - \sigma_y(\gamma_i)}{VH(\beta) - VH(\gamma_i)} VH(\gamma_i) + \frac{\sigma_y(\gamma_i) VH(\beta) - \sigma_y(\gamma_i) VH(\gamma)}{VH(\beta) - VH(\gamma)}
\]

where \(\sigma_y(\alpha_i)\), \(\sigma_y(\gamma_i)\) and \(\sigma_y(\beta)\) are the yield stresses of original ferrite, original austenite, and bainite layer, respectively, VH(αi) and VH(γi) are the Vickers microhardness of each layer in α and γ regions respectively, and VH(αi), VH(γi) and VH(β) are the Vickers microhardness of original ferrite, original austenite, and bainite layer, respectively.

If it is assumed that the stress-strain curve of each layer obeys the Holloman relation, the imposed stress to each element at yield strain of bainitic layer may be given as\cite{8};

\[
\sigma'_y = \sigma_y(\alpha_i) \left( \frac{\varepsilon_y}{\varepsilon(\alpha_i)} \right)^{n(\alpha_i)}
\]
\[
\sigma_i'' = \sigma_0(\gamma_i) \left[ \frac{\varepsilon_i}{\varepsilon(\gamma_i)} \right]^{n(\gamma_i)} \tag{4}
\]

where \(\sigma_i'\) and \(\sigma_i''\) are the imposed stresses to each layer in \(\alpha\) and \(\gamma\) regions at yield strain of bainitic layer, respectively, \(\varepsilon_\beta\) is the yield strain of the bainitic layer, and \(n(\alpha_i)\) and \(n(\gamma_i)\) are the strain-hardening coefficient of each layer in \(\alpha\) and \(\gamma\) regions, respectively.

It is assumed that the strain hardening coefficient of each layer in the studied composite obeys exponential function, therefore\[^{[8]}\]:

\[
n(\alpha_i) = n(\beta) \exp \left( \frac{X - X_\beta}{X_\beta - X_\alpha} \right) \ln \left( \frac{n(\beta)}{n(\alpha)} \right) \tag{5}
\]

\[
n(\gamma_i) = n(\beta) \exp \left( \frac{X - X_\beta}{X_\beta - X_\gamma} \right) \ln \left( \frac{n(\beta)}{n(\gamma)} \right) \tag{6}
\]

where \(n(\alpha)\), \(n(\gamma)\) and \(n(\beta)\) are the strain-hardening coefficient of original ferrite, original austenite and bainite layer at distance \(X\), respectively, \(X_\alpha\), \(X_\gamma\) and \(X_\beta\) are the distances of original ferrite, original austenite and bainite layer respectively, and \(\varepsilon(\alpha_i)\) and \(\varepsilon(\gamma_i)\) in Equations (3) and (4) are defined as the yield strain of each layer in \(\alpha\) and \(\gamma\) regions, respectively.

By considering the boundary conditions, at \(i = 1\), \(\varepsilon(\alpha_i) = \varepsilon(\alpha)\); at \(i = m_\alpha\), \(\varepsilon(\alpha_i) = \varepsilon(\beta)\)

Similarly, at \(i = 1\), \(\varepsilon(\gamma_i) = \varepsilon(\gamma)\); at \(i = m_\gamma\), \(\varepsilon(\gamma_i) = \varepsilon(\beta)\)

where \(\varepsilon(\alpha)\), \(\varepsilon(\gamma)\), and \(\varepsilon(\beta)\) are the yield strain of original ferrite, original austenite and bainite layer, respectively.

The yield strain of each layer in \(\alpha\) and \(\gamma\) regions of the composite is as follows\[^{[8]}\]:

\[
\varepsilon_y(\alpha_i) = \frac{1}{E} \left[ \frac{\sigma_y(\beta) - \sigma_y(\alpha)}{V_H(\beta) - V_H(\alpha)} \right] V_H(\alpha_i) + \frac{\sigma_y(\alpha).V_H(\beta) - \sigma_y(\beta).V_H(\alpha)}{V_H(\beta) - V_H(\alpha)} \tag{7}
\]

\[
\varepsilon_y(\gamma_i) = \frac{1}{E} \left[ \frac{\sigma_y(\gamma) - \sigma_y(\beta)}{V_H(\gamma) - V_H(\beta)} \right] V_H(\gamma_i) + \frac{\sigma_y(\beta).V_H(\gamma) - \sigma_y(\gamma).V_H(\beta)}{V_H(\gamma) - V_H(\beta)} \tag{8}
\]

where \(E\) is the Young modulus.

Therefore the stress-strain curve of each layer could be determined. Furthermore, an exponential relation may be assumed for a linking curve between fracture point of the stress-strain curves of original ferrite and single-phase bainite, and original austenite and single-phase bainite (i.e. the dashed connecting curves shown in Fig. 6) with suitable boundary conditions as follows\[^{[10]}\]. The assumption of exponential function is based on the previous works\[^{[11,12]}\):

\[
\sigma_{ts}(\alpha_i) = \sigma_{ts}(\beta) \exp \left( \frac{\varepsilon_{ts}(\alpha_i) - \varepsilon_{ts}(\beta)}{\varepsilon_{ts}(\beta) - \varepsilon_{ts}(\alpha)} \right) \ln \left( \frac{\sigma_{ts}(\beta)}{\sigma_{ts}(\alpha)} \right) \tag{9}
\]

\[
\sigma_{ts}(\gamma_i) = \sigma_{ts}(\beta) \exp \left( \frac{\varepsilon_{ts}(\gamma_i) - \varepsilon_{ts}(\beta)}{\varepsilon_{ts}(\beta) - \varepsilon_{ts}(\gamma)} \right) \ln \left( \frac{\sigma_{ts}(\beta)}{\sigma_{ts}(\gamma)} \right) \tag{10}
\]

where \(\sigma_{ts}(\alpha)\), \(\sigma_{ts}(\gamma)\) and \(\sigma_{ts}(\beta)\) are the ultimate tensile stress of original ferrite, original austenite and bainite layer, respectively, and \(\varepsilon_{ts}(\alpha)\), \(\varepsilon_{ts}(\gamma)\) and \(\varepsilon_{ts}(\beta)\) are the ultimate tensile strain of original ferrite, original austenite and bainite layer, respectively.

By combining the Hollomon relation for each layer and Eqs. (9) and (10) or both for \(\alpha\) and \(\gamma\) region or both, tensile stress and strain of each layer could be determined. Therefore, by integrating the Hollomon relation corresponding to each stress-strain curve in the range of yield strain to tensile strain, in addition to the area under elastic portion of stress-strain curve and presuming Hollomon relation is valid up to the true ultimate tensile stress (i.e. fracture point)\[^{[8]}\], the total area under stress-strain curve of each layer could be determined as follows\[^{[10]}\):

\[
S = \frac{\varepsilon_y(\alpha_i).\sigma_y(\alpha)}{2} + \int_{\varepsilon_y(\alpha_i)}^{\varepsilon_{ts}(\alpha)} K \varepsilon^{n(\alpha)} d\varepsilon \tag{11}
\]
where $S$ is the area under stress-strain curve of each layer, and $K$ is the Holloman constant. Subsequently
the corresponding area under stress-strain curve of that layer utilizing a suitable function (i.e. an exponential
function). Therefore by applying suitable boundary conditions we have,

$$ J_{IC}(\alpha_i) = J_{IC}(\beta) \exp \left( \frac{S - S_B}{S_B - S_{\alpha}} \cdot \ln \left( \frac{J_{IC}(\beta)}{J_{IC}(\alpha_i)} \right) \right) $$

(13)

$$ J_{IC}(\gamma_i) = J_{IC}(\beta) \exp \left( \frac{S - S_B}{S_B - S_{\gamma}} \cdot \ln \left( \frac{J_{IC}(\beta)}{J_{IC}(\gamma_i)} \right) \right) $$

(14)

where $J_{IC}(\alpha_i)$ and $J_{IC}(\gamma_i)$ are the fracture toughness of each layer in $\alpha$ and $\gamma$ regions respectively, $J_{IC}(\alpha)$, $J_{IC}(\gamma)$ and $J_{IC}(\beta)$ are the fracture toughness of original ferrite, original austenite and bainite layer, respectively, and $S_{\alpha}$, $S_{\gamma}$ and $S_B$ are the corresponding area under stress-strain curve of original ferrite, original austenite and bainite layer, respectively.

Finally by utilizing the rule of mixtures we are able to calculate fracture toughness of $\alpha\beta\gamma$ composite as,

$$ J_{IC}^{\alpha\beta\gamma} = \sum_{i=1}^{m_{\alpha}} \frac{J_{IC}(\beta)}{S_{\alpha}} \left[ S_{\beta} \left( \frac{J_{IC}(\beta)}{S_{\beta}} - \frac{J_{IC}(\alpha)}{S_{\alpha}} \right) + J_{IC}(\alpha) - J_{IC}(\beta) \right] \cdot V(\alpha_i) + J_{IC}(\beta) \cdot V(\beta) + \sum_{i=1}^{m_{\gamma}} \frac{J_{IC}(\beta)}{S_{\gamma}} \left[ S_{\beta} \left( \frac{J_{IC}(\beta)}{S_{\beta}} - \frac{J_{IC}(\gamma)}{S_{\gamma}} \right) + J_{IC}(\gamma) - J_{IC}(\beta) \right] \cdot V(\gamma_i) $$

(15)

where $V(\beta)$ is the thickness of the bainitic layer.

By taking account of the thickness of each layer as 10 micron, fracture toughness of $\alpha\beta\gamma$ composite has been
determined and it is shown in Table 4. Seemingly there is an excellent agreement between the experimental
result and that acquired from the proposed model.

With the same procedure explained above, fracture toughness of the other composites was also modeled.
The obtained fracture toughness value from the mathematical model for all composites is also given in Table 4.
Obviously there is a good agreement between results. Previous work showed that Charpy impact energy of
$\gamma$M$\gamma$ composite determined by the rule of mixtures suffer a relatively large deviation from experimental data.
In fact, in Charpy impact test due to high strain rate of the dynamic loading condition the martensite layer
with the poorest impact energy (i.e. fastest crack growth rate) controls the crack propagation. It seems that the exceedingly large difference of impact energies between austenite and martensite layer leading to a sharp gradient at $\gamma$/M interface causes the deviation in the results. Therefore it may be concluded that the accelerating effect of crack growth in brittle martensitic phase may have synergistic effect on crack growth in the neighboring austenite layer. However, in the case of static fracture toughness test of $J_{IC}$, the martensite layer does not disrupt the fracture toughness of $\gamma$M$\gamma$ composite likewise. Thus, one may anticipate that fracture toughness of the functionally graded steels obeys the rule of mixtures even in the presence of martensite layer as shown by the analytical model.

Although there is a good agreement between the experimental and analytical results for all composites, the
analytical results may depend on the exponential function presumed for the relation between variations of
fracture toughness with the corresponding area under stress-strain curve in the graded regions. This is performed for this reason that the other functions may release better results than exponential one. To validate the readability of the utilized function, two other functions have been examined for correlating the variation of fracture toughness with respect to the corresponding area under stress-strain curve. The first function was homographic function for which by applying the suitable boundary conditions for $\alpha\beta\gamma$, $J_{IC}$ could be calculated as;

$$ J_{IC}^{\alpha\beta\gamma} = \sum_{i=1}^{m_{\alpha}} \frac{J_{IC}(\beta)}{S_{\alpha}} \left[ S_{\beta} \left( \frac{J_{IC}(\beta)}{S_{\beta}} - \frac{J_{IC}(\alpha)}{S_{\alpha}} \right) + J_{IC}(\alpha) - J_{IC}(\beta) \right] \cdot V(\alpha_i) + J_{IC}(\beta) \cdot V(\beta) + \sum_{i=1}^{m_{\gamma}} \frac{J_{IC}(\beta)}{S_{\gamma}} \left[ S_{\beta} \left( \frac{J_{IC}(\beta)}{S_{\beta}} - \frac{J_{IC}(\gamma)}{S_{\gamma}} \right) + J_{IC}(\gamma) - J_{IC}(\beta) \right] \cdot V(\gamma_i) $$

(16)
The other was power-law function\(^{(9,10)}\) and again by applying the suitable boundary conditions for \(\alpha\beta\gamma\), \(J_{IC}\) could be calculated as,

\[
J_{IC}\alpha\beta\gamma = \sum_{i=1}^{m} J_{IC}(\beta) \cdot \left( \frac{S}{\sigma_{\beta}} \right) ^{\ln \left( \frac{d_{IC}(\beta)}{d_{IC}(\beta)_{\text{ref}}} \right)} \cdot V(\alpha_i) + J_{IC}(\beta) \cdot V(\beta) + \sum_{i=1}^{n} J_{IC}(\beta) \cdot \left( \frac{S}{\sigma_{\gamma}} \right) ^{\ln \left( \frac{d_{IC}(\beta)}{d_{IC}(\beta)_{\text{ref}}} \right)} \cdot V(\gamma_i)
\]

For the other composites, the two other functions have been examined and no evident improvement in fracture toughness of the composites by utilizing the new functions was seen.

Table 4 shows that in \(\alpha\beta\gamma\) composite, displacement of the bainite layer toward the alpha region (reduction of alpha region thickness) causes greater fracture toughness of the composite; however the deviation of the theoretical results from experimental ones becomes greater. Although it seems astonishing, its reason should be evaluated. One may consider that this difference is related to the position of the layers. For the edge layers, plane stress condition is indefeasible while the central layers are in plane strain condition. Formation of plane stress condition may result in the following contour.

Table 4 shows that the displacement of martensite layer in \(\gamma M\gamma\) composite causes different state in \(J_{IC}\) of the composites. When the martensite layer is situated in the center of \(\gamma M\gamma\) composite, the deviation of theoretical results from the experimental ones becomes smaller. 1 mm displacement of the martensite layer towards the left side of \(\gamma M\gamma\) composite means that 1 mm of the left-edge austenite region is translocated to the right edge of the specimen. In the other words, \(J_{IC}\) of the edge layer should be modified according to the layers position.

Table 4 shows that the displacement of martensite layer in \(\gamma M\gamma\) composite causes different state in \(J_{IC}\) of the composites. When the martensite layer is situated in the center of \(\gamma M\gamma\) composite, the deviation of theoretical results from the experimental ones becomes smaller. 1 mm displacement of the martensite layer towards the left side of \(\gamma M\gamma\) composite means that 1 mm of the left-edge austenite region is translocated to the right edge of the specimen. In the other words, the change is not applied on the composite layers. Once more this confirms the position dependency of the layers’ \(J_{IC}\).

Even in a monolithic material, the energy absorbed by the middle part and edges of the specimen is not equal (Fig. 7)\(^{(17)}\). In fact, the plane strain condition in the center of the specimen results in dissimilar energy absorption with respect to the edges of the specimen which are in plane stress condition. To distinguish these two conditions, it is assumed that the energy absorbed by each part is related to the plastic zone size of the specimen. In the present functionally graded steels, when the tougher layers (i.e. austenite and bainite) are situated in the specimen edges, the total required force becomes greater. In the other words, \(J_{IC}\) of the edge layer should be modified according to the layers position.

\[
r_{p} = \frac{1}{4\pi} \left( \frac{K_{I}}{\sigma_{y}} \right)^{2} \left[ 1 + \cos \theta \left( 1 - 2\nu^{2} \right) + \frac{3}{2} \sin^{2} \theta \right]
\]

where \(r_{p}\) is the plastic zone size, \(K_{I}\) is the mode I stress intensity factor, \(\sigma_{y}\) is the yield stress of material, \(\nu\) is the Poisson’s ratio, and \(\theta\) denotes the angular position in polar coordinate. While in the specimen’s edges, plane stress condition may result in the following contour \(^{(18)}\):

\[
r_{p} = \frac{1}{4\pi} \left( \frac{K_{I}}{\sigma_{y}} \right)^{2} \left( 1 + \cos \theta + \frac{3}{2} \sin^{2} \theta \right)
\]

If it is assumed that the un-cracked part of the monolithic specimen ahead of the crack is composed of \(n\) element, fracture toughness of the monolithic specimen is the mean value of the fracture toughness of these \(n\) layers,

\[
J_{IC} = \sum_{i=1}^{n} J_{IC,i}
\]

where \(J_{IC,i}\) is the fracture toughness of each element. In fact, \(J_{IC,i}\) is the \(J_{IC}\) of the specimen multiplied by a ratio of the plane stress condition to the plane strain condition, \(f_{i}\), i.e.,

\[
J_{IC,i} = J_{IC} \cdot f_{i}
\]

\(f_{i}\) depends on the element position and the closer the element to the specimen edges, the higher the \(f_{i}\). To

\[\text{Fig. 7 Plastic zone ahead of crack in a plate of finite thickness. At the edges of the plate the stress state is close to plane stress. In the center of a sufficiently thick plate the stress state approximates to plane strain}^{[17]}\]
where \( x \) in Eq. (15), the modified distance with a combination of plane stress and plane strain conditions, and utilizing exponential function used was obtained as 1.43 mm from the edge of the specimen. In the other words, by applying Eq. (27) to a certain could be obtained as;

\[
\text{thickness.}
\]

By supposing that the variation of the elements’ plastic zone size from the edges of the specimen to the center of that specimen obeys an exponential function, by applying the suitable boundary conditions and assuming that the origin of the Cartesian coordinate is at the center of the specimen, we have,

\[
f_i = f_{\text{plane-stress}} \cdot \exp \left( \frac{2x}{W} \ln \left( \frac{f_{\text{plane-stress}}}{f_{\text{plane-strain}}} \right) \right)
\]

where \( x \) is the distance of each element with respect to the center of the specimen, and \( W \) is the specimen thickness.

To calculate \( f_{\text{plane-stress}} \) and \( f_{\text{plane-strain}} \), we have;

\[
J_{IC} = \sum_{i=1}^{n} J_{IC,i} = J_{IC} \cdot \sum_{i=1}^{n} f_i
\]

To satisfy Eq. (24), we must have,

\[
\sum_{i=1}^{n} f_i = 1
\]

By applying the mean value theorem, it could be written,

\[
\int_{0}^{\frac{W}{2}-t} f_i \, dx = \frac{W}{2} - t
\]

where \( \frac{W}{2}-t \) represent the thickness in which there exist a combination of plane stress and plane strain condition.

By substituting Eq. (23) into Eq. (20), \( f_i \) could be determined as,

\[
f_i = \exp \left( \frac{0.65x}{W} \right)
\]

Although the relation between the net plane stress and net plane strain condition has been obtained, the layer in which net plane strain condition finishes and plane stress conditions starts gradually is unknown. Thus, that layer was selected in which by applying Eq. (27), no differences between experimental and analytical results has been observed. For \( \alpha\beta\gamma \) composite, the distance of the layer in which plane stress condition starts was obtained as 1.43 mm from the edge of the specimen. In the other words, by applying Eq. (27) to a certain distance with a combination of plane stress and plane strain conditions, and utilizing exponential function used in Eq. (15), the modified \( J_{IC} \) of \( \alpha\beta\gamma \) composite with the bainite layer situated at the center of the specimen, could be obtained as;

<table>
<thead>
<tr>
<th>Table 5</th>
<th>Experimental and modified theoretical fracture toughness, ( J_{IC} ), of the studied composites (kJ/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha\beta\gamma )</td>
<td>Distance of ( \beta ) from ( \gamma ) edge ( d_{\beta} = 0 \quad d_{\beta} = 1 \quad d_{\beta} = 3 \quad d_{\beta} = 4.7 \quad d_{\beta} = 5 \quad d_{\beta} = 7 \quad d_{\beta} = 9.4 )</td>
</tr>
<tr>
<td>Experimental result</td>
<td>12.4</td>
</tr>
<tr>
<td>Analytical result</td>
<td>+3.0</td>
</tr>
<tr>
<td>Deviation</td>
<td>8.1</td>
</tr>
</tbody>
</table>

| \( \gamma M \gamma \) | Distance of \( M \) from \( \gamma \) edge \( d_{M} = 0 \quad d_{M} = 1 \quad d_{M} = 3 \quad d_{M} = 4.2 \quad d_{M} = 5 \) |
| Experimental result | 9.6 | 10.1 | 10.5 | 11 | 10.8 |
| Analytical result | 10.2 | 10.7 | 11.1 | 11.7 | 11.5 |
| Deviation | +5.9 | +5.8 | +6.0 | +5.9 | +6.1 |
\[
\sum_{i=1}^{n_x} J_{\text{IC}}(\beta) \cdot \exp \left( \frac{S - S_\beta}{S_\beta - S_\gamma} \cdot \ln \left( \frac{J_{\text{IC}}(\beta)}{J_{\text{IC}}(\gamma)} \right) \right) \cdot \exp \left( 0.65 \frac{i}{n} \right) \cdot V(\gamma_i) + \\
\sum_{i=1}^{n_x} J_{\text{IC}}(\beta) \cdot \exp \left( \frac{S - S_\beta}{S_\beta - S_\gamma} \cdot \ln \left( \frac{J_{\text{IC}}(\beta)}{J_{\text{IC}}(\gamma)} \right) \right) \cdot V(\gamma_i)
\] (28)

4. Conclusions

(1) Fracture toughness \((J_{\text{IC}})\) of various FGSs depends on the thickness and number of layers.
(2) \(J_{\text{IC}}\) of the produced composites is between \(J_{\text{IC}}\) of AISI 1020 and AISI 316 steels.
(3) The mathematical model which has been proposed based on the rule of mixtures for determination \(J_{\text{IC}}\) of the composites show good agreement with those of experiments for all composites. However, the model has been modified by a function which depends on the layers position.
(4) For the edge layers, net plane stress condition was supposed and a gradual alteration from net plane stress condition to net plane strain condition was applied to approximately 1.5 mm thickness from the edge of the specimens. The results obtained after modification show better compatibility with the experimental ones.

REFERENCES